[2][a] $\frac{dy}{dx} = \frac{y^4 - x^2y^2 - 2x^4}{xy^3 - 3x^4}$

ALL UNDERLINED ITEMS $2\frac{1}{2}$ POINTS UNLESS OTHERWISE LABELLED

$(xy^3 - 3x^4)\,dy = (y^4 - x^2y^2 - 2x^4)\,dx$

$\underbrace{(2x^4 + x^2y^2 - y^4)}_{M}\,dx + \underbrace{(xy^3 - 3x^4)}_{N}\,dy = 0$ $3\frac{1}{2}$

$M(tx,ty) = 2(tx)^4 + (tx)^2(ty)^2 - (ty)^4 = t^4(2x^4 + x^2y^2 - y^4)$ $3\frac{1}{2}$

$= t^4 M(x,y)$

$N(tx,ty) = (tx)(ty)^3 - 3(tx)^4 = t^4(xy^3 - 3x^4) = t^4 N(x,y)$ $3\frac{1}{2}$

BOTH COEFFICIENTS HOMOGENEOUS OF ORDER 4

DE IS HOMOGENEOUS

LET $y = vx \longrightarrow dy = v\,dx + x\,dv$ AND $v = \frac{y}{x}$

$3\frac{1}{2}$ $(2x^4 + v^2x^4 - v^4x^4)\,dx + (v^3x^4 - 3x^4)(v\,dx + x\,dv) = 0$

$(2 + v^2 - v^4)\,dx + (v^3 - 3)(v\,dx + x\,dv) = 0$

$(2 + v^2 - \cancel{v^4} + \cancel{v^4} - 3v)\,dx + x(v^3 - 3)\,dv = 0$

$(v^2 - 3v + 2)\,dx + x(v^3 - 3)\,dv = 0$

$(v^2 - 3v + 2)\,dx = -x(v^3 - 3)\,dv$

$3\frac{1}{2}$ $\int -\frac{1}{x}\,dx = \int \frac{v^3 - 3}{v^2 - 3v + 2}\,dv$

CHECKPOINT: SEPARABLE

$-\int \frac{1}{x}\,dx = \int \left(v + 3 + \frac{2}{v-1} + \frac{5}{v-2}\right) dv$ $3\frac{1}{2}$

LONG DIVISION + PARTIAL FRACTIONS WORK SHOWN BELOW

$-\ln|x| + C = \frac{1}{2}v^2 + 3v + 2\ln|v-1| + 5\ln|v-2|$ $6$

$= \frac{1}{2}\left(\frac{y}{x}\right)^2 + 3\left(\frac{y}{x}\right) + 2\ln\left|\frac{y}{x} - 1\right| + 5\ln\left|\frac{y}{x} - 2\right|$

$= \frac{y^2}{2x^2} + \frac{3y}{x} + 2\ln\left|\frac{y-x}{x}\right| + 5\ln\left|\frac{y-2x}{x}\right|$

$-\ln|x| + C = \frac{y^2 + 6xy}{2x^2} + 2\ln|y-x| + 5\ln|y-2x| - 7\ln|x|$

$$\frac{y^2+6xy}{2x^2}+2\ln|y-x|+5\ln|y-2x|=6\ln|x|+C$$

2

$$e^{\frac{y^2+6xy}{2x^2}}(y-x)^2(y-2x)^5=Cx^6$$

3

$$(y-x)^{4x^2}(y-2x)^{10x^2}e^{y^2+6xy}=Cx^{12x^2}$$

2

$$\begin{array}{r} v + 3 \\ v^2-3v+2 \overline{\big)\, v^3 \qquad\qquad\qquad -3} \\ \underline{v^3 - 3v^2 + 2v} \\ 3v^2 - 2v - 3 \\ \underline{3v^2 - 9v + 6} \\ 7v - 9 \end{array}$$

(6)

$$\frac{7v-9}{v^2-3v+2} = \boxed{\frac{\cancel{A}\,2}{v-1} + \frac{\cancel{B}\,5}{v-2}}$$

$$\underline{7v-9 = A(v-2) + B(v-1)}$$

$v=1$: $\quad -2 = -A \longrightarrow \underline{A=2}$

$v=2$: $\quad \underline{5 = B}$

SANITY CHECK: $v=3$

LEFT $\quad \dfrac{21-9}{9-9+2} = \dfrac{12}{2} = 6$

RIGHT $\quad \dfrac{2}{2} + \dfrac{5}{1} = 1+5 = 6$

[2][b] $\frac{dr}{d\theta} = \frac{(r^3\sin\theta - \csc\theta)r}{\sec\theta} = r^4\sin\theta\cos\theta - r\cot\theta$ 6

$\frac{dr}{d\theta} + (\cot\theta)r = (\sin\theta\cos\theta)r^4 \longrightarrow$ BERNOULLI $n=4$

$v = r^{1-4} = r^{-3} \longrightarrow \frac{dv}{d\theta} = -3r^{-4}\frac{dr}{d\theta}$

$-3r^{-4}\frac{dr}{d\theta} - 3(\cot\theta)r^{-3} = -3\sin\theta\cos\theta$ $3\frac{1}{2}$

$\frac{dv}{d\theta} - 3(\cot\theta)v = -3\sin\theta\cos\theta$ CHECKPOINT: LINEAR

$\mu = e^{-3\int\cot\theta\, d\theta} = e^{-3\ln|\sin\theta|} = (\sin\theta)^{-3} = \csc^3\theta$

$u = \sin\theta \longrightarrow du = \cos\theta\, d\theta$

$\int\frac{du}{u}$

$3\frac{1}{2}$ $\csc^3\theta\,\frac{dv}{d\theta} - 3(\csc^3\theta\cot\theta)v = -3\cos\theta\csc^2\theta = -3\csc\theta\cot\theta$

CHECKPOINT: $(\csc^3\theta)' = 3\csc^2\theta(-\csc\theta\cot\theta)$

$= -3\csc^3\theta\cot\theta$

$(\csc^3\theta)v = \int -3\csc\theta\cot\theta\, d\theta = 3\csc\theta + C$

$r^{-3} = v = 3\sin^2\theta + C\sin^3\theta$ $3\frac{1}{2}$

$2^{-3} = 3(-\frac{1}{2})^2 + C(-\frac{1}{2})^3$

$\frac{1}{8} = \frac{3}{4} - \frac{C}{8}$

$1 = 6 - C$

$C = 5$

$r = (3\sin^2\theta + 5\sin^3\theta)^{-\frac{1}{3}}$ $3\frac{1}{2}$

[3] $\underbrace{(2x^{m+5}e^{(k+1)t}-3x^{m+1}e^{kt})}_{M}dt+\underbrace{(6x^{m+1}e^{(k+3)t}-4x^{m}e^{kt})}_{N}dx=0$ 5

$M_x = 2(m+5)x^{m+4}e^{(k+1)t}-3(m+1)x^{m}e^{kt}$ 5

$= N_t = 6(k+3)x^{m+1}e^{(k+3)t}-4kx^{m}e^{kt}$ 5

$2(m+5)=0 \rightarrow m=-5$

$6(k+3)=0 \rightarrow k=-3$

$-3(m+1)=-3(-4)=12=-4k=-4(-3)=12$ ✓

$\mu = x^{-5}e^{-3t}$

5 $\underbrace{(2e^{-2t}-3x^{-4}e^{-3t})}_{P}dt+\underbrace{(6x^{-4}-4x^{-5}e^{-3t})}_{Q}dx=0$

$P_x = 12x^{-5}e^{-3t} = Q_t = -4x^{-5}e^{-3t}(-3) = 12x^{-5}e^{-3t}$ EXACT

$f = \int(2e^{-2t}-3x^{-4}e^{-3t})dt = -e^{-2t}+x^{-4}e^{-3t}+C(x)$ $3\frac{1}{2}$

$f_x = -4x^{-5}e^{-3t}+C'(x)$

$= 6x^{-4}-4x^{-5}e^{-3t}$

$C'(x) = 6x^{-4}$ CHECKPOINT: FUNCTION OF x ONLY

$C(x) = -2x^{-3}$

$f = -e^{-2t}+x^{-4}e^{-3t}-2x^{-3}=C$ MULTIPLY BY $-x^4e^{3t}$

$x^4e^{t}-1+2xe^{3t}=Cx^4e^{3t}$ $3\frac{1}{2}$